OKLAHOMA STATE UNIVERSITY

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN/MAE 5713 Linear Systems
Spring 2011
Final Exam

Choose any four out of five problems.
Please specify which four listed below to be graded:
1)_____; 2)_____; 3)_____; 4)_____;

Name: ______________________________

E-Mail Address:_______________________________________

Problem 1:

Find a minimal *controllable* canonical form realization (i.e., its simulation diagram and state space representation) for the following system described by

$$H(s) = \begin{bmatrix} \dfrac{2s}{s^3 + 6s^2 + 11s + 6} \\ \dfrac{s^2 + 2s + 2}{s^4 + 6s^3 + 9s^2 + 4s} \end{bmatrix}.$$

<u>Problem 2</u>:

Let

$$S = \{x \in \Re^3 \mid x = \alpha[1 \quad 0 \quad 2] + \beta[2 \quad 0 \quad 4],\ \alpha, \beta \in \Re\},$$

find the orthogonal complement space of S, $S^{\perp} (\subset \Re^3)$, and determine an orthonormal basis and dimension for $S^{\perp}$. For $x = [1 \quad 2 \quad 3] (\in \Re^3)$, find its direct sum representation (i.e., x_1 and x_2) of $x = x_1 \oplus x_2$, such that $x_1 \in S, x_2 \in S^{\perp}$.

Problem 3:

Find the state transition matrix of

$$\dot{x} = \begin{bmatrix} -\sin t & 0 \\ 0 & -\cos t \end{bmatrix} x .$$

Problem 4:
Let

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find e^{At}.

Problem 5:
Consider

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

and its adjoint system

$$\begin{aligned}\dot{z} &= -A^T z + C^T v \\ w &= B^T z\end{aligned}.$$

Show

$$H(s) = -H_a^T(-s),$$

where $H(s)$ and $H_a(s)$ are their transfer function matrices, respectively.